Να αποδείξετε ότι η παράγωγος της συνάρτησης

$$f(x) = \operatorname{Arcsin}\left(\frac{2x+3}{5}\right) + 2\operatorname{Arctan}\sqrt{\frac{1-x}{4+x}}\ , \ \forall x \in$$

Είναι ίση με το μηδέν.

<u>ΛΥΣΗ</u>

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{2x+3}{5}\right)^2}} \cdot \left(\frac{2x+3}{5}\right)' + 2\,\frac{1}{1+\frac{1-x}{4+x}} \cdot \left(\sqrt{\frac{1-x}{4+x}}\right)' =$$

$$= \frac{2}{5} \cdot \frac{1}{\sqrt{1-\left(\frac{2x+3}{5}\right)^2}} + \cancel{2} \cdot \frac{1}{1+\frac{1-x}{4+x}} \cdot \frac{1}{\cancel{2}\sqrt{\frac{1-x}{4+x}}} \cdot \left(\frac{1-x}{4+x}\right)' =$$

$$= \frac{2}{\cancel{5}} \cdot \frac{\cancel{5}}{\sqrt{5^2-(2x+3)^2}} + \frac{\cancel{(4+x)}}{\cancel{5}} \cdot \sqrt{\frac{4+x}{1-x}} \cdot \left(-\frac{\cancel{5}}{(4+x)^{\cancel{2}}}\right) =$$

$$= \frac{2}{\sqrt{5^2-(2x+3)^2}} - \sqrt{\frac{4+x}{1-x}} \cdot \frac{1}{4+x} =$$

$$= \frac{2}{\sqrt{5^2-(2x+3)^2}} - \frac{(4+x)^{1/2}}{(1-x)^{1/2}\cdot(4+x)} =$$

$$= \frac{2}{\sqrt{(2-2x)(8+2x)}} - \frac{1}{(1-x)^{1/2}\cdot(4+x)^{1/2}} =$$

$$= \frac{\cancel{2}}{\cancel{\sqrt{2}}\cdot\sqrt{1-x}\cdot\cancel{\sqrt{2}}\cdot\sqrt{4+x}} - \frac{1}{(1-x)^{1/2}\cdot(4+x)^{1/2}} =$$

$$= \cancel{\frac{1}{(1-x)^{1/2}\cdot(4+x)^{1/2}} - \frac{1}{(1-x)^{1/2}\cdot(4+x)^{1/2}}} = 0.$$